

# First Evidence for $\cos 2\beta > 0$ and Resolution of the Cabibbo-Kobayashi-Maskawa Quark-Mixing Unitarity Triangle Ambiguity

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 (Received 17 April 2018; revised manuscript received 22 August 2018; published 26 December 2018)

We present first evidence that the cosine of the  $CP$ -violating weak phase  $2\beta$  is positive, and hence exclude trigonometric multifold solutions of the Cabibbo-Kobayashi-Maskawa (CKM) Unitarity Triangle using a time-dependent Dalitz plot analysis of  $B^0 \rightarrow D^{(*)}h^0$  with  $D \rightarrow K_S^0\pi^+\pi^-$  decays, where  $h^0 \in \{\pi^0, \eta, \omega\}$  denotes a light unflavored and neutral hadron. The measurement is performed combining the final data sets of the *BABAR* and Belle experiments collected at the  $\Upsilon(4S)$  resonance at the asymmetric-energy  $B$  factories PEP-II at SLAC and KEKB at KEK, respectively. The data samples contain  $(471 \pm 3) \times 10^6 B\bar{B}$  pairs recorded by the *BABAR* detector and  $(772 \pm 11) \times 10^6 B\bar{B}$  pairs recorded by the Belle detector. The results of the measurement are  $\sin 2\beta = 0.80 \pm 0.14(\text{stat}) \pm 0.06(\text{syst}) \pm 0.03(\text{model})$  and  $\cos 2\beta = 0.91 \pm 0.22(\text{stat}) \pm 0.09(\text{syst}) \pm 0.07(\text{model})$ . The result for the direct measurement of the angle  $\beta$  of the CKM Unitarity Triangle is  $\beta = [22.5 \pm 4.4(\text{stat}) \pm 1.2(\text{syst}) \pm 0.6(\text{model})]^\circ$ . The measurement assumes no direct  $CP$  violation in  $B^0 \rightarrow D^{(*)}h^0$  decays. The quoted model uncertainties are due to the composition of the  $D^0 \rightarrow K_S^0\pi^+\pi^-$  decay amplitude model, which is newly established by performing a Dalitz plot amplitude analysis using a high-statistics  $e^+e^- \rightarrow c\bar{c}$  data sample.  $CP$  violation is observed in  $B^0 \rightarrow D^{(*)}h^0$  decays at the level of 5.1 standard deviations. The significance for  $\cos 2\beta > 0$  is 3.7 standard deviations. The trigonometric multifold solution  $\pi/2 - \beta = (68.1 \pm 0.7)^\circ$  is excluded at the level of 7.3 standard deviations. The measurement resolves an ambiguity in the determination of the apex of the CKM Unitarity Triangle.

DOI: [10.1103/PhysRevLett.121.261801](https://doi.org/10.1103/PhysRevLett.121.261801)

In the standard model (SM) of electroweak interactions, the only source of  $CP$  violation is the irreducible complex phase in the three-family Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1]. The *BABAR* and Belle experiments discovered  $CP$  violation in the  $B$  meson system [2–5]. In particular, by time-dependent  $CP$  violation measurements of the “gold plated” decay mode  $B^0 \rightarrow J/\psi K_S^0$  and other decays mediated by  $\bar{b} \rightarrow \bar{c}c\bar{s}$  transitions [6,7], *BABAR* and Belle precisely determined the parameter  $\sin 2\beta \equiv \sin 2\phi_1$  (*BABAR* uses the notation  $\beta$  and Belle uses  $\phi_1$ ; hereinafter  $\beta$  is used), where the angle  $\beta$  of the CKM Unitarity Triangle is defined as  $\arg[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*]$  and  $V_{ij}$  denotes a CKM matrix element. (In this Letter, the inclusion of charge-conjugated decay modes is implied unless otherwise stated.) Inferring the  $CP$ -violating weak phase  $2\beta$  from these measurements of  $\sin 2\beta$  leads to the trigonometric twofold ambiguity,  $2\beta$  and  $\pi - 2\beta$  (a fourfold ambiguity in  $\beta$ ), and therefore to an ambiguity on the CKM Unitarity Triangle. This ambiguity can be resolved by also measuring  $\cos 2\beta$ , which is experimentally accessible in  $B$  meson decay modes involving multibody final states such as  $B^0 \rightarrow J/\psi K_S^0\pi^0$  [8,9],  $B^0 \rightarrow D^{*+}D^{*-}K_S^0$

[10,11],  $B^0 \rightarrow K_S^0 K^+ K^-$  [12,13],  $B^0 \rightarrow K_S^0\pi^+\pi^-$  [14,15], and  $B^0 \rightarrow D^{(*)}h^0$  with  $D \rightarrow K_S^0\pi^+\pi^-$  decays (abbreviated as  $B^0 \rightarrow [K_S^0\pi^+\pi^-]^{(*)}_D h^0$ ) [16–18]. However, no previous single measurement has been sufficiently sensitive to establish the sign of  $\cos 2\beta$ , to resolve the ambiguity without further assumptions.

The decays  $B^0 \rightarrow D^{(*)}h^0$ , with  $D \rightarrow K_S^0\pi^+\pi^-$  and  $h^0 \in \{\pi^0, \eta, \omega\}$  denoting a light neutral hadron, provide an elegant way to access  $\cos 2\beta$  [19]. The  $B^0 \rightarrow D^{(*)}h^0$  decay is predominantly mediated by CKM-favored  $\bar{b} \rightarrow \bar{c}u\bar{d}$  tree amplitudes. Additional contributions from CKM-disfavored  $\bar{b} \rightarrow \bar{u}c\bar{d}$  tree amplitudes that carry different weak phases are suppressed by  $|V_{ub}V_{cd}^*/V_{cb}V_{ud}^*| \approx 0.02$  relative to the leading amplitudes and can be neglected at the experimental sensitivity of the presented measurement. The  $D \rightarrow K_S^0\pi^+\pi^-$  decay exhibits complex interference structures that receive resonant and nonresonant contributions to the three-body final state from a rich variety of intermediate  $CP$  eigenstates and quasi-flavor-specific decays. Knowledge of the variations on the relative strong phase as a function of the three-body Dalitz plot phase space enables measurements of both  $\sin 2\beta$  and  $\cos 2\beta$  from the time evolution of the  $B^0 \rightarrow [K_S^0\pi^+\pi^-]^{(*)}_D h^0$  multibody final state.

Assuming no  $CP$  violation in  $B^0 - \bar{B}^0$  mixing and no direct  $CP$  violation, the rate of the  $B^0 \rightarrow [K_S^0\pi^+\pi^-]^{(*)}_D h^0$  decays is proportional to

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$$\frac{e^{-\frac{|\Delta t|}{\tau_{B^0}}}}{2} \{ [|\mathcal{A}_{\bar{D}^0}|^2 + |\mathcal{A}_{D^0}|^2] - q(|\mathcal{A}_{\bar{D}^0}|^2 - |\mathcal{A}_{D^0}|^2) \cos(\Delta m_d \Delta t) + 2q\eta_{h^0}(-1)^L \text{Im}(e^{-2i\beta} \mathcal{A}_{D^0} \mathcal{A}_{\bar{D}^0}^*) \sin(\Delta m_d \Delta t) \}, \quad (1)$$

where  $\Delta t$  denotes the proper-time interval between the decays of the two  $B$  mesons produced in the  $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B^0 \bar{B}^0$  event, and  $q = +1$  ( $-1$ ) represents the  $b$ -flavor content when the accompanying  $B$  meson is tagged as a  $B^0$  ( $\bar{B}^0$ ). The parameters  $\tau_{B^0}$  and  $\Delta m_d$  are the neutral  $B$  meson lifetime and the  $B^0 - \bar{B}^0$  oscillation frequency, respectively. The symbols  $\mathcal{A}_{D^0} \equiv \mathcal{A}(M_{K_S^0 \pi^-}^2, M_{K_S^0 \pi^+}^2)$  and  $\mathcal{A}_{\bar{D}^0} \equiv \mathcal{A}(M_{K_S^0 \pi^+}^2, M_{K_S^0 \pi^-}^2)$  denote the  $D^0$  and  $\bar{D}^0$  decay amplitudes as functions of the Lorentz-invariant Dalitz plot variables  $M_{K_S^0 \pi^-}^2 \equiv (p_{K_S^0} + p_{\pi^-})^2$  and  $M_{K_S^0 \pi^+}^2 \equiv (p_{K_S^0} + p_{\pi^+})^2$ , where the symbol  $p_i$  represents the four-momentum of a final state particle  $i$ . The factor  $\eta_{h^0}$  is the  $CP$  eigenvalue of  $h^0$ . The quantity  $L$  is the orbital angular momentum of the  $Dh^0$  or  $D^*h^0$  system. The last term in Eq. (1) can be rewritten as

$$\text{Im}(e^{-2i\beta} \mathcal{A}_{D^0} \mathcal{A}_{\bar{D}^0}^*) = \text{Im}(\mathcal{A}_{D^0} \mathcal{A}_{\bar{D}^0}^*) \cos 2\beta - \text{Re}(\mathcal{A}_{D^0} \mathcal{A}_{\bar{D}^0}^*) \sin 2\beta, \quad (2)$$

which allows  $\sin 2\beta$  and  $\cos 2\beta$  to be treated as independent parameters.

Measurements of  $\sin 2\beta$  and  $\cos 2\beta$  in  $B^0 \rightarrow D^{(*)}h^0$  with  $D \rightarrow K_S^0 \pi^+ \pi^-$  decays are experimentally challenging. The branching fractions of the  $B$  and  $D$  meson decays are low [ $\mathcal{O}(10^{-4})$  and  $\mathcal{O}(10^{-2})$ , respectively], and the neutral particles in the final state lead to large backgrounds and low reconstruction efficiencies. In addition, a detailed Dalitz plot amplitude model or other experimental knowledge of the relative strong phase in the three-body  $D$  meson decay is required. Previous measurements of these decays performed separately by *BABAR* and Belle were not sufficiently sensitive to establish  $CP$  violation [16–18], obtaining results far outside of the physical region of the parameter space [16], and using different Dalitz plot amplitude models [16,17], which complicates the combination of individual results.

In this Letter, we present measurements of  $\sin 2\beta$  and  $\cos 2\beta$  from a time-dependent Dalitz plot analysis of  $B^0 \rightarrow D^{(*)}h^0$  with  $D \rightarrow K_S^0 \pi^+ \pi^-$  decays that combines the final data samples collected by the *BABAR* and Belle experiments, totaling  $1.1 \text{ ab}^{-1}$  collected at the  $\Upsilon(4S)$  resonance. The combined approach enables unique experimental sensitivity to  $\cos 2\beta$  by increasing the available data sample and by applying common assumptions and the same Dalitz plot amplitude model simultaneously to the data collected by both experiments. As part of the analysis, an improved  $D \rightarrow K_S^0 \pi^+ \pi^-$  Dalitz plot amplitude model is

obtained from high-statistics  $e^+e^- \rightarrow c\bar{c}$  data. This allows the propagation of the model uncertainties to the results on  $\sin 2\beta$  and  $\cos 2\beta$  obtained in  $B^0 \rightarrow D^{(*)}h^0$  with  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$  decays in a straightforward way. In the following, the extraction of the  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$  Dalitz plot amplitude model parameters from Belle  $e^+e^- \rightarrow c\bar{c}$  data is summarized. Thereafter, the time-dependent Dalitz plot analysis of the  $B$  meson decay combining *BABAR* and Belle data is described. A more detailed description of the analysis is provided in Ref. [20].

To measure the  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$  decay amplitudes, we use a data sample of  $924 \text{ fb}^{-1}$  recorded at or near the  $\Upsilon(4S)$  and  $\Upsilon(5S)$  resonances with the Belle detector [21] at the asymmetric-energy  $e^+e^-$  collider KEKB [22]. This gives a large sample of  $D$  mesons enabling precise measurement of the decay amplitudes, so there is no benefit to be gained from including the equivalent *BABAR* data. The decays  $D^{*+} \rightarrow D^0 \pi^+$  with  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$  and  $K_S^0 \rightarrow \pi^+ \pi^-$  are reconstructed, and the flavor of the neutral  $D$  meson is identified as  $D^0$  ( $\bar{D}^0$ ) by the positive (negative) charge of the slow pion  $\pi_s^+$  emitted from the  $D^{*+}$  decay. Charged pion candidates are formed from reconstructed tracks, and the selection requirements described in Refs. [23,24] are applied to  $K_S^0$  candidates. To reject background originating from  $B$  meson decays, a requirement of  $p^*(D^{*+}) > 2.5(3.1) \text{ GeV}/c$  for candidates reconstructed from  $\Upsilon(4S)$  [ $\Upsilon(5S)$ ] data is applied, where  $p^*$  denotes the momentum evaluated in the  $e^+e^-$  center-of-mass (c.m.) frame. Events are selected by the  $D^0$  candidate mass  $M_{D^0}$  and the  $D^{*+} - D^0$  mass difference  $\Delta M$ , and a yield of  $1217300 \pm 2000$  signal decays is obtained by a two-dimensional unbinned maximum-likelihood fit to the  $M_{D^0}$  and  $\Delta M$  distributions [20].

Similar to previous  $D^0 - \bar{D}^0$  oscillation analyses and measurements of the Unitarity Triangle angle  $\gamma$  [25] by *BABAR*, Belle, and LHCb [26–29], the  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$  decay amplitude is parametrized as

$$\begin{aligned} \mathcal{A}(M_{K_S^0 \pi^-}^2, M_{K_S^0 \pi^+}^2) = & \sum_{r \neq (K\pi/\pi\pi)_{L=0}} a_r e^{i\phi_r} \mathcal{A}_r(M_{K_S^0 \pi^-}^2, M_{K_S^0 \pi^+}^2) \\ & + F_1(M_{\pi^+ \pi^-}^2) + \mathcal{A}_{K\pi_{L=0}}(M_{K_S^0 \pi^-}^2) \\ & + \mathcal{A}_{K\pi_{L=0}}(M_{K_S^0 \pi^+}^2). \end{aligned} \quad (3)$$

The symbols  $a_r$  and  $\phi_r$  represent the magnitude and phase of the  $r$ th intermediate quasi-two-body amplitude  $\mathcal{A}_r$  contributing to the  $P$ - and  $D$ -waves. These amplitudes are parametrized using an isobar ansatz [30] by relativistic Breit-Wigner (BW) propagators with mass-dependent widths, Blatt-Weisskopf penetration factors [31], and Zemach tensors for the angular distributions [32]. The following intermediate two-body resonances are included: the Cabibbo-favored  $K^*(892)^- \pi^+$ ,  $K_2^*(1430)^- \pi^+$ ,  $K^*(1680)^- \pi^+$ ,  $K^*(1410)^- \pi^+$  channels; the doubly Cabibbo-suppressed  $K^*(892)^+ \pi^-$ ,  $K_2^*(1430)^+ \pi^-$ ,

$K^*(1410)^+\pi^-$  modes; and the  $CP$  eigenstates  $K_S^0\rho(770)^0$ ,  $K_S^0\omega(782)$ ,  $K_S^0f_2(1270)$ , and  $K_S^0\rho(1450)^0$ . The symbol  $F_1$  denotes the amplitude for the  $\pi\pi S$ -wave using the  $K$ -matrix formalism in the  $P$ -vector approximation with four physical poles [33,34]. The symbol  $\mathcal{A}_{K\pi L=0}$  represents the amplitude for the  $K\pi S$ -wave using the LASS parametrization [35], which combines a BW for the  $K_0^*(1430)^\pm$  with a coherent nonresonant contribution governed by an effective range and a phase shift.

The  $D^0 \rightarrow K_S^0\pi^+\pi^-$  decay amplitude model parameters are determined by an unbinned maximum-likelihood Dalitz fit performed for events in the signal region of the flavor-tagged  $D^0$  sample. The probability density function (p.d.f.) for the signal is constructed from Eq. (3) with a correction to account for reconstruction efficiency variations over the Dalitz plot phase space due to experimental acceptance effects [36], and an additional term to account for wrong flavor identifications of  $D$  mesons. In addition, the likelihood function contains a p.d.f. for the background that is constructed from the distributions taken from the  $M_{D^0}$  and  $\Delta M$  data sidebands. The  $a_r$  and  $\phi_r$  parameters for each resonance are floated in the fit and measured relative to the  $K_S^0\rho(770)^0$  amplitude, which is fixed to  $a_{K_S^0\rho(770)^0} = 1$  and  $\phi_{K_S^0\rho(770)^0} = 0^\circ$ . The masses and widths of the resonances are fixed to the world averages [37] except for those of the  $K^*(892)$  and  $K_0^*(1430)$ , which are floated to improve the fit quality. The LASS parameters and several parameters in the  $K$ -matrix are floated in the fit.

The results of the Dalitz fit are summarized in Table III of Ref. [20]. The data distributions and projections of the fit are shown in Fig. 1. By a two-dimensional  $\chi^2$  test, a reduced  $\chi^2$  of 1.05 is obtained for 31 272 degrees of freedom based on statistical uncertainties only, indicating a relatively good quality of the fit [26–28,38,39].

The time-dependent Dalitz plot analysis of  $B^0 \rightarrow D^{(*)}h^0$  with  $D \rightarrow K_S^0\pi^+\pi^-$  decays is performed using data samples containing  $471 \times 10^6 B\bar{B}$  pairs recorded with the *BABAR* detector [40,41] at the asymmetric-energy  $e^+e^-$  (3.1 on 9 GeV) collider PEP-II [42] and  $772 \times 10^6 B\bar{B}$  pairs recorded with the Belle detector [21] at the asymmetric-energy  $e^+e^-$  (3.5 on 8 GeV) collider KEKB [22] collected at the  $\Upsilon(4S)$ .

The light neutral hadron  $h^0$  is reconstructed in the decay modes  $\pi^0 \rightarrow \gamma\gamma$ ,  $\eta \rightarrow \gamma\gamma$  and  $\pi^+\pi^-\pi^0$ , and  $\omega \rightarrow \pi^+\pi^-\pi^0$ . Neutral  $D$  mesons are reconstructed in the decay mode  $D \rightarrow K_S^0\pi^+\pi^-$ , and neutral  $D^*$  mesons are reconstructed in the decay mode  $D^* \rightarrow D\pi^0$ . The decay modes  $B^0 \rightarrow D\pi^0$ ,  $D\eta$ ,  $D\omega$ ,  $D^*\pi^0$ , and  $D^*\eta$ , where sufficient signal yields are reconstructed, are included in the analysis. The selection requirements applied to the reconstructed candidates are summarized in Ref. [20].

The  $B^0 \rightarrow D^{(*)}h^0$  yields are determined by three-dimensional unbinned maximum-likelihood fits to the

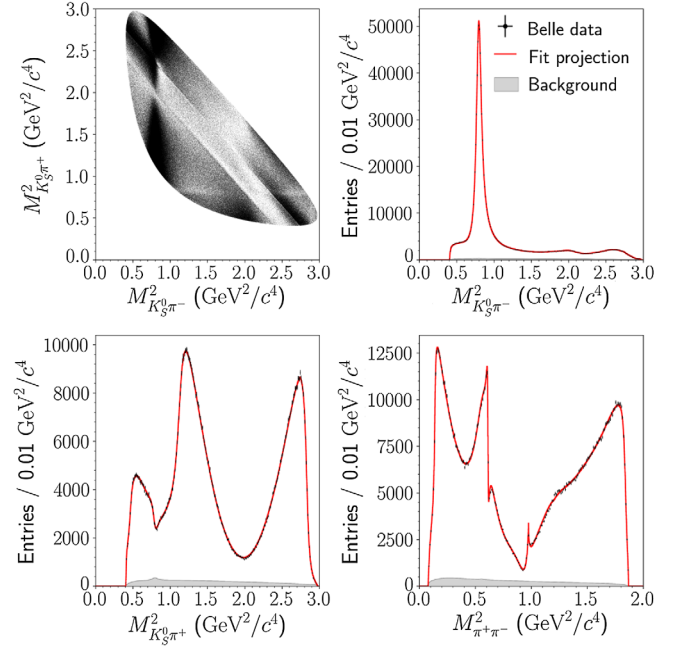


FIG. 1. The Dalitz plot data distributions (points with error bars) for  $D^0 \rightarrow K_S^0\pi^+\pi^-$  from  $D^{*+} \rightarrow D^0\pi_s^+$  decays reconstructed from Belle  $e^+e^- \rightarrow c\bar{c}$  data, and projections of the Dalitz fit. The red solid lines show the projections of the total fit function including background, and the grey regions show projections of the background.

distributions of the observables  $M'_{bc}$ ,  $\Delta E$ , and  $C'_{NN_{out}}$ . The beam-energy-constrained mass  $M'_{bc}$  defined in Ref. [43] is computed from the beam energy  $E_{beam}^*$  in the c.m. frame, the  $D^{(*)}$  candidate momenta, and the  $h^0$  candidate direction of flight. The quantity  $M'_{bc}$  provides an observable that is insensitive to possible correlations with the energy difference  $\Delta E = E_B^* - E_{beam}^*$  that can be induced by energy mismeasurements for particles detected in the electromagnetic calorimeters, e.g., caused by shower leakage effects. The variable  $C'_{NN_{out}}$  defined in Ref. [44] is constructed from the output of a neural network multivariate classifier trained on event shape information based on a combination of 16 modified Fox-Wolfram moments [45,46] to identify background originating from  $e^+e^- \rightarrow q\bar{q}$  ( $q \in \{u, d, s, c\}$ ) continuum events. The fit model accounts for contributions from  $B^0 \rightarrow D^{(*)}h^0$  signal decays, cross-feed from partially reconstructed  $B^0 \rightarrow D^*h^0$  decays, background from partially reconstructed  $B^+ \rightarrow \bar{D}^{(*)0}\rho^+$  decays, combinatorial background from  $B\bar{B}$  decays, and background from continuum events. In total, a  $B^0 \rightarrow D^{(*)}h^0$  signal yield of  $1129 \pm 48$  events in the *BABAR* data sample and  $1567 \pm 56$  events in the Belle data sample is obtained. The signal yields are summarized in Table IV of Ref. [20]. The  $M'_{bc}$ ,  $\Delta E$ , and  $C'_{NN_{out}}$  data distributions and fit projections are shown in Fig. 2.

The time-dependent Dalitz plot analysis follows the technique established in the previous combined *BABAR*



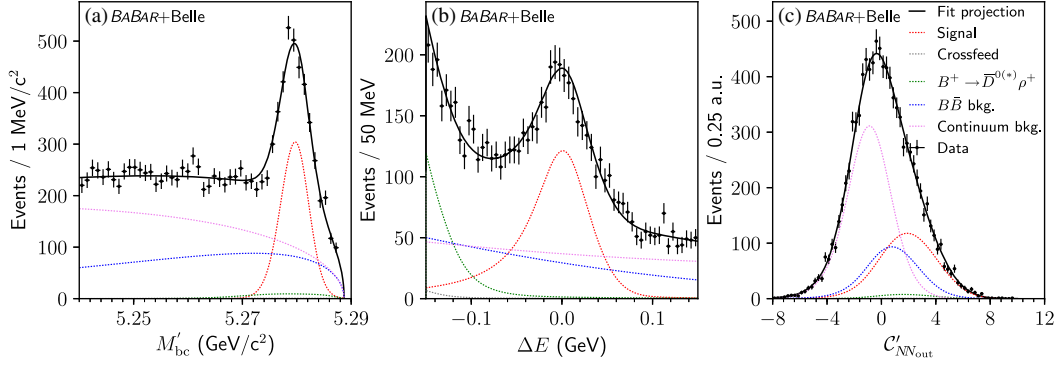


FIG. 2. Data distributions for (a)  $M'_{bc}$ , (b)  $\Delta E$ , and (c)  $C'_{NNout}$  (points with error bars) for the *BABAR* and Belle data samples combined. The solid black lines represent projections of the total fit function, and the colored dotted lines show the signal and background components of the fit as indicated in the legend. In plotting the  $M'_{bc}$ ,  $\Delta E$ , and  $C'_{NNout}$  distributions, each of the other two observables are required to satisfy  $M'_{bc} > 5.272 \text{ GeV}c^2$ ,  $|\Delta E| < 100 \text{ MeV}$ , or  $0 < C'_{NNout} < 8$  to select signal-enhanced regions.

+Belle time-dependent  $CP$  violation measurement of  $\bar{B}^0 \rightarrow D_{CP}^{(*)} h^0$  decays [24]. The measurement is performed by maximizing the log-likelihood function constructed from the events reconstructed from *BABAR* and Belle data [20]. The measurement includes all events used in the previous  $M'_{bc}$ ,  $\Delta E$ , and  $C'_{NNout}$  fits. In the log-likelihood function, the p.d.f.'s are functions of the experimental flavor-tagged proper-time interval and Dalitz plot distributions for the signal and background components. The signal p.d.f.s are constructed from Eqs. (1) and (2) convolved with experiment-specific resolution functions to account for the finite vertex resolution [6,47] and including the effect of incorrect flavor assignments [6,48]. The p.d.f.'s for the proper-time interval distributions of the combinatorial background from  $B\bar{B}$  decays and background from continuum events account for background from nonprompt and prompt particles convolved with effective resolution functions. The partially reconstructed  $B^0 \rightarrow D^* h^0$  decays are modeled by the signal p.d.f. with a different set of parameters to account for this cross-feed contribution, and the background from partially reconstructed  $B^+ \rightarrow \bar{D}^{(*)0} \rho^+$  decays is parametrized by an exponential p.d.f. convolved with the same resolution functions as used for the signal.

In the fit, the parameters  $\tau_{B^0}$ ,  $\tau_{B^+}$ , and  $\Delta m_d$  are fixed to the world averages [49], and the Dalitz plot amplitude model parameters are fixed to the results of the  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$  Dalitz plot fit described above. The signal and background fractions are evaluated on an event-by-event basis from the three-dimensional fit of the  $M'_{bc}$ ,  $\Delta E$ , and  $C'_{NNout}$  observables. The only free parameters are  $\sin 2\beta$  and  $\cos 2\beta$ , and the results are

$$\begin{aligned} \sin 2\beta &= 0.80 \pm 0.14(\text{stat}) \pm 0.06(\text{syst}) \pm 0.03(\text{model}), \\ \cos 2\beta &= 0.91 \pm 0.22(\text{stat}) \pm 0.09(\text{syst}) \pm 0.07(\text{model}). \end{aligned} \quad (4)$$

The second quoted uncertainty is the experimental systematic error, and the third is due to the  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$  decay amplitude model. The evaluation of these uncertainties is described in detail in Ref. [20]. The linear correlation between  $\sin 2\beta$  and  $\cos 2\beta$  is 5.1%. The result deviates by less than 1.0 standard deviation from the trigonometric constraint given by  $\sin^2 2\beta + \cos^2 2\beta = 1$ .

A separate fit is performed to measure directly the angle  $\beta$  using the signal p.d.f. constructed from Eq. (1), and the result is

$$\beta = [22.5 \pm 4.4(\text{stat}) \pm 1.2(\text{syst}) \pm 0.6(\text{model})]^\circ. \quad (5)$$

The proper-time interval distributions and projections of the fit for  $\sin 2\beta$  and  $\cos 2\beta$  are shown in Fig. 3 for two different regions of the  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$  phase space. Figure 3(a) shows a region predominantly populated by  $CP$  eigenstates,  $B^0 \rightarrow [K_S^0 \rho(770)^0]_D^{(*)} h^0$ . For these decays, interference emerges between the amplitude for direct decays of neutral  $B$  mesons into these final states and those following  $B^0 - \bar{B}^0$  oscillations. The time evolution exhibits mixing-induced  $CP$  violation governed by the  $CP$ -violating weak phase  $2\beta$ , which manifests as a sinusoidal oscillation in the signal yield asymmetry. Figure 3(b) shows a region predominantly populated by quasi-flavor-specific decays,  $B^0 \rightarrow [K^*(892)^\pm \pi^\mp]_D^{(*)} h^0$ . For these decays, the time evolution exhibits  $B^0 - \bar{B}^0$  oscillations governed by the oscillation frequency,  $\Delta m_d$ , which appears as an oscillation proportional to  $\cos(\Delta m_d \Delta t)$  in the corresponding asymmetry.

The measurement procedure is validated by various cross-checks. The  $B^0 \rightarrow \bar{D}^{(*)0} h^0$  decays with the CKM-favored  $\bar{D}^0 \rightarrow K^+ \pi^-$  decay have very similar kinematics and background composition as  $B^0 \rightarrow D^{(*)} h^0$  with  $D \rightarrow K_S^0 \pi^+ \pi^-$  decays and provide a high-statistics control sample. Using the same analysis approach, the time-dependent  $CP$  violation measurement of the control sample



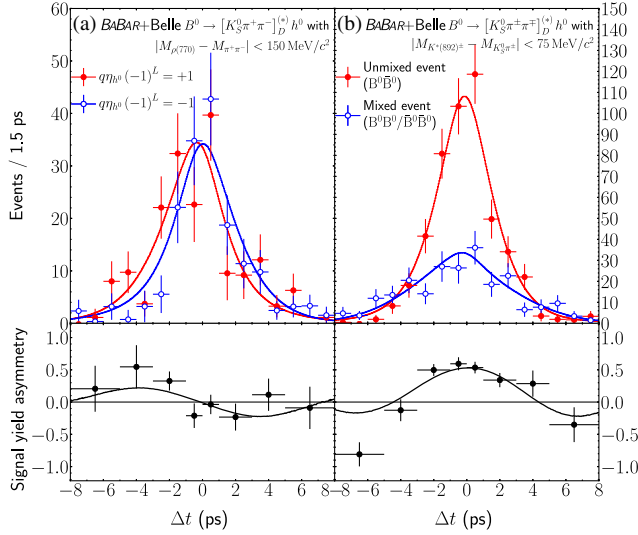


FIG. 3. Distributions of the proper-time interval (data points with error bars) and the corresponding asymmetries for  $B^0 \rightarrow D^{(*)}h^0$  candidates associated with high-quality flavor tags for two different regions of the  $D \rightarrow K_S^0 \pi^+ \pi^-$  phase space and for the *BABAR* and Belle data samples combined. The background has been subtracted using the  $\mathcal{S}$ Plot technique [50], with weights obtained from the fit presented in Fig. 2.

results in mixing-induced and direct  $CP$  violation parameters consistent with zero, in agreement with the assumption of negligible  $CP$  violation for these flavor-specific decays. Measurements of the neutral  $B$  meson lifetime for  $B^0 \rightarrow D^{(*)}h^0$  with  $D \rightarrow K_S^0 \pi^+ \pi^-$  decays, and for the control sample without flavor-tagging applied, yield  $\tau_{B^0} = [1.500 \pm 0.052(\text{stat})]$  ps and  $\tau_{B^0} = [1.535 \pm 0.028(\text{stat})]$  ps, respectively, which are in agreement with the world average  $\tau_{B^0} = (1.520 \pm 0.004)$  ps [49]. In addition, we have performed all measurements for data separated by experiment yielding consistent results [20].

The significance of the results is determined by a likelihood-ratio approach that accounts for the experimental systematic uncertainties and the Dalitz plot amplitude model uncertainties by convolution of the likelihood curves. The measurement of  $\sin 2\beta$  agrees within 0.7 standard deviations with the world average of  $\sin 2\beta = 0.691 \pm 0.017$  [49] obtained from more precise measurements using  $\bar{b} \rightarrow \bar{c}c\bar{s}$  transitions. The measurement of  $\cos 2\beta$  excludes the hypothesis of  $\cos 2\beta \leq 0$  at a  $p$ -value of  $2.5 \times 10^{-4}$ , which corresponds to a significance of 3.7 standard deviations, providing the first evidence for  $\cos 2\beta > 0$ . The measurement of  $\beta$  excludes the hypothesis of  $\beta = 0^\circ$  at a  $p$ -value of  $3.6 \times 10^{-7}$ , which corresponds to a significance of 5.1 standard deviations. Hence, we report an observation of  $CP$  violation in  $B^0 \rightarrow D^{(*)}h^0$  decays. The result for  $\beta$  agrees well with the preferred solution of the Unitarity Triangle, which is  $(21.9 \pm 0.7)^\circ$ , if computed from the world average of  $\sin 2\beta = 0.691 \pm 0.017$  [49]. The measurement excludes the second solution of

$\pi/2 - \beta = (68.1 \pm 0.7)^\circ$  at a  $p$ -value of  $2.31 \times 10^{-13}$ , corresponding to a significance of 7.3 standard deviations. Therefore, the present measurement resolves an ambiguity in the determination of the apex of the CKM Unitarity Triangle.

In summary, we combine the final *BABAR* and Belle data samples, totaling an integrated luminosity of more than  $1 \text{ ab}^{-1}$  collected at the  $\Upsilon(4S)$  resonance, and perform a time-dependent Dalitz plot analysis of  $B^0 \rightarrow D^{(*)}h^0$  with  $D \rightarrow K_S^0 \pi^+ \pi^-$  decays. We report the world's most precise measurement of the cosine of the  $CP$ -violating weak phase  $2\beta$  and obtain the first evidence for  $\cos 2\beta > 0$ . The measurement directly excludes the trigonometric multifold solution of  $\pi/2 - \beta = (68.1 \pm 0.7)^\circ$  without any assumptions, and thus resolves an ambiguity related to the CKM Unitarity Triangle parameters. An observation of  $CP$  violation in  $B^0 \rightarrow D^{(*)}h^0$  decays is reported. The measurement assumes no direct  $CP$  violation in  $B^0 \rightarrow D^{(*)}h^0$  decays.

The  $B^0 \rightarrow D^{(*)}h^0$  decays studied by the combined *BABAR* and Belle approach provide a probe for the  $CP$ -violating weak phase  $2\beta$  that is theoretically more clean than the “gold plated” decay modes mediated by  $\bar{b} \rightarrow \bar{c}c\bar{s}$  transitions [51]. Therefore,  $B^0 \rightarrow D^{(*)}h^0$  decays can provide a new and complementary SM reference for  $2\beta$  at the experimental precision achievable by the future high-luminosity  $B$  factory experiment Belle II [52].

We thank the PEP-II and KEKB groups for the excellent operation of the accelerators. The *BABAR* experiment acknowledges the substantial dedicated effort from the computing organizations for their support. The collaborating institutions wish to thank SLAC for its support and kind hospitality. The Belle experiment wishes to acknowledge the KEK cryogenics group for efficient solenoid operations; and the KEK computer group, the NII, and PNNL/EMSL for valuable computing and SINET5 network support. This work was supported by MEXT, JSPS and Nagoya's TLPRC (Japan); ARC (Australia); FWF (Austria); NSERC (Canada); NSFC and CCEPP (China); MSM (Czechia); CEA and CNRS-IN2P3 (France); BMBF, CZF, DFG, EXC153, and VS (Germany); DST (India); INFN (Italy); MOE, MSIP, NRF, RSRI, FLRFAS project and GSDC of KISTI (Korea); FOM (The Netherlands); NFR (Norway); MNiSW and NCN (Poland); MES and RFAAE (Russia); ARRS (Slovenia); IKERBASQUE and MINECO (Spain); SNSF (Switzerland); MOE and MOST (Taiwan); STFC (United Kingdom); BSF (USA-Israel); and DOE and NSF (USA). Individuals have received support from the Marie Curie EIF (European Union) and the A. P. Sloan Foundation (USA).

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